2/20/20

SOA "Severity" Problems

Remark: (Use Fact)

$$(X-d)_+ = \begin{cases} 0 & \text{if } X<d \\ X-d & \text{if } X>d \end{cases} \qquad X \wedge d = \begin{cases} X & \text{if } X<d \\ d & \text{if } X>d \end{cases}$$

$$\Longrightarrow (X-d)_+ + (X \wedge d) = X$$

84)

96)

28)

87)

89)

100)

See next pages

84. A health plan implements an incentive to physicians to control hospitalization under which the physicians will be paid a bonus B equal to c times the amount by which total hospital claims are under 400 $(0 \le c \le 1)$.

The effect the incentive plan will have on underlying hospital claims is modeled by assuming that the new total hospital claims will follow a two-parameter Pareto distribution with $\alpha = 2$ and $\theta = 300$.

$E(B) = 100$

Calculate c.

(A) 0.44

(B) 0.48

(C) 0.52

(D) 0.56

(E) 0.60

Let X = rvr the total hospital claims

$X \sim$ 2-Pareto $(\alpha = 2, \theta = 300)$

$$B = c \cdot (400 - X)_+$$

$$100 = c \cdot E[(400 - X)_+]$$

$$E[(X-d)_+] = E[X] - E[X \wedge d]$$

$$E[(400 - X)_+] = E[400] - E[\underbrace{400 \wedge X}_{= X \wedge 400}]$$

$$E[X \wedge 400] \overset{\text{P.4}}{\underset{\text{Tables}}{=}} \frac{300}{2-1}\left[1 - \left(\frac{300}{400+300}\right)^1\right]$$

$$\therefore E[(400 - X)_+] = 400 - 300\left(1 - \frac{3}{7}\right)$$

$$\Rightarrow c = .4375 \quad \text{(A)}$$

95. The number of claims in a period has a geometric distribution with mean 4. The amount of each claim X follows $\Pr(X = x) = 0.25, \quad x = 1, 2, 3, 4$, The number of claims and the claim amounts are independent. S is the aggregate claim amount in the period.

Calculate $F_S(3)$.

(A) 0.27

(B) 0.29

(C) 0.31

(D) 0.33

(E) 0.35

96. Insurance agent Hunt N. Quotum will receive no annual bonus if the ratio of incurred losses to earned premiums for his book of business is 60% or more for the year. If the ratio is less than 60%, Hunt's bonus will be a percentage of his earned premium equal to 15% of the difference between his ratio and 60%. Hunt's annual earned premium is 800,000.

$L =$ Incurred losses are distributed according to the Pareto distribution, with $\theta = 500{,}000$ and $\alpha = 2$.

$L \sim$ 2-Pareto $(\alpha = 2, \theta = 500000)$

Calculate the expected value of Hunt's bonus.

(A) 13,000

(B) 17,000

(C) 24,000

(D) 29,000

(E) 35,000

$$R = \frac{L}{800000} \Rightarrow R \sim \text{2-Pareto}\left(\alpha = 2, \theta = \tfrac{5}{8}\right)$$

$$B = 800000(.15) \cdot (.6 - R)_+$$
$$= 120000\,(.6 - R_+)$$
$$\Rightarrow E[B] = 120000 \cdot E[(.6 - R)_+]$$

$$E[(.6 - R)_+] = E[.6] - E[R \wedge .6]$$
$$= .6 - \tfrac{5}{8}\left(1 - \left(\frac{5/8}{.6 + 5/8}\right)\right)$$

$\therefore E[B] = 35{,}265$ E

28. You are given:

Claim Size (X)	Number of Claims
(0, 25]	25
(25, 50]	28
(50, 100]	15
(100, 200]	6

$/ \Sigma = 74$

Assume a uniform distribution of claim sizes within each interval.

Estimate $E(X^2) - E[(X \wedge 150)^2]$. $= E[X^2 - (X \wedge 150)^2]$

$$(X \wedge 150)^2 = \begin{cases} X^2 & \text{if } X < 150 \\ 150^2 & \text{if } X > 150 \end{cases}$$

(A) Less than 200

(B) At least 200, but less than 300

(C) At least 300, but less than 400

(D) At least 400, but less than 500

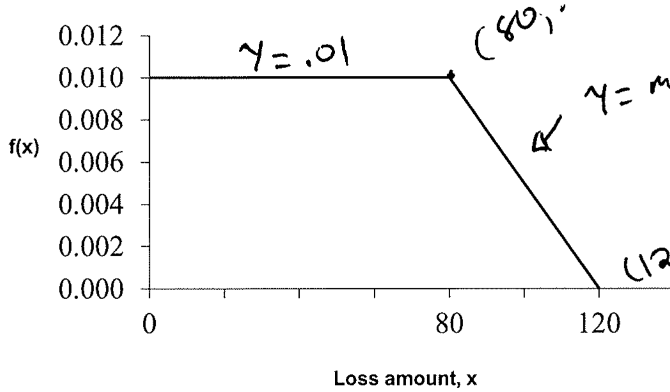
(E) At least 500

$$\therefore X^2 - (X \wedge 150)^2 = \begin{cases} 0 & \text{if } X < 150 \\ X^2 - 150^2 & \text{if } X > 150 \end{cases}$$

Given $X > 150$, $f_X(x) = \frac{1}{100}$

$$E[X^2 - (X \wedge 150)^2] = \left[\int_{150}^{200} (x^2 - 150^2) \cdot \frac{1}{100}\, dx\right] \cdot \frac{6}{74}$$

87. The graph of the density function for losses is:



$y = .01$ $\quad (80, .01)$

$y = mx + b$ $\quad m = -\frac{.01}{40} = -.00025$

$(120, 0) \to 0 = -.00025(120) + b$

$b = .03$

$y = .03 - .00025x$

Calculate the loss elimination ratio for an ordinary deductible of 20.

(A) 0.20

(B) 0.24

(C) 0.28

(D) 0.32

(E) 0.36

$$LER = \frac{E[X \wedge d]}{E[X]}$$

$$E[X] = \int_0^{120} x \cdot f(x)\, dx$$

$$E[X \wedge 20] = \int_0^{20} x \cdot f(x)\, dx + \int_{20}^{120} 20 \cdot f(x)\, dx$$

$$f(x) = \begin{cases} .01 & 0 < x < 80 \\ .03 - .00025x & 80 < x < 120 \end{cases}$$

89. You are given:

(i) Losses follow an exponential distribution with the same mean in all years.

(ii) The loss elimination ratio this year is 70%.

(iii) The ordinary deductible for the coming year is 4/3 of the current deductible.

Calculate the loss elimination ratio for the coming year.

(A) 70%

(B) 75%

(C) 80%

(D) 85%

(E) 90%

From Tables, $E[X \wedge x] = \theta(1 - e^{-x/\theta})$ (for Exp Distribution)

d = deductible amount for this year

$\Rightarrow \frac{4}{3}d$ = deductible amount for coming year

$$.7 = \frac{E[X \wedge d]}{E[X]} = \frac{\cancel{\theta}(1 - e^{-d/\theta})}{\cancel{\theta}} \Longrightarrow e^{-d/\theta} = .3$$

$$LER^{new} = \frac{E[X \wedge 4/3 d]}{E[X]} = \frac{\cancel{\theta}(1 - e^{-4/3 d/\theta})}{\cancel{\theta}} = 1 - (e^{-d/\theta})^{4/3} = 1 - .3^{4/3}$$

90. Actuaries have modeled auto windshield claim frequencies. They have concluded that the number of windshield claims filed per year per driver follows the Poisson distribution with parameter λ, where λ follows the gamma distribution with mean 3 and variance 3.

Calculate the probability that a driver selected at random will file no more than 1 windshield claim next year.

(A) 0.15

(B) 0.19

(C) 0.20

(D) 0.24

(E) 0.31

100. The unlimited severity distribution for claim amounts under an auto liability insurance policy is given by the cumulative distribution:

$$F(x) = 1 - 0.8e^{-0.02x} - 0.2e^{-0.001x}, \quad x \geq 0$$

The insurance policy pays amounts up to a limit of 1000 per claim.

Calculate the expected payment under this policy for one claim. $E[X \wedge 1000] = ?$

(A) 57

(B) 108

(C) 166

(D) 205

(E) 240

X is a 80%/20% mixture of Y/W

$Y \sim Exp(\theta = 50)$

$W \sim Exp(\theta = 1000)$

101. The random variable for a loss, X, has the following characteristics:

x	$F(x)$	$E(X \wedge x)$
0	0.0	0
100	0.2	91
200	0.6	153
1000	1.0	331

Calculate the mean excess loss for a deductible of 100.

(A) 250

(B) 300

(C) 350

(D) 400

(E) 450